

# **IVS Memorandum**

**15 July 2008**

**“Impact of different turbulence, clock  
and white noise parameters on VLBI2010  
simulation results with OCCAM Kalman  
Filter”**

**Jörg Wresnik, Andrea Pany, Johannes Böhm**

## Monte Carlo Simulator

The Monte Carlo Simulator used for the V2C simulations in Vienna creates the artificial observations based on realistic properties of the wet zenith delays and clocks, and of the predicted instrumental error of the VLBI2010 observing system. The observed group delay minus computed group delay ( $o-c$ ) can be described as follows:

$$o - c = (zwd_2 \cdot mfw_2(e_2) + cl_2) - (zwd_1 \cdot mfw_1(e_1) + cl_1) + wn_{Bsl} \quad (1)$$

In Eq. (1),  $zwd_{1,2}$  are simulated zenith wet delays based on the turbulence model by Tobias Nilsson et al. (2007),  $cl_{1,2}$  are simulated clock values modeled as a random walk plus integrated random walk at station 1 and 2 of each observation (see Boehm et al. 2007).  $mfw_{1,2}(e_{1,2})$  are the wet mapping functions for the elevation angle  $e_{1,2}$ , and they are assumed to be error-less in these studies. For each baseline observation, an additional white noise  $wn_{Bsl}$  is added to model the instrumental errors of station 1 and 2. The Monte Carlo simulator, implemented in OCCAM, imports wet zenith delay values from the turbulence model, creates clock values for each station and epoch, and adds white noise for each observation.

## Impact of turbulence parameters on VLBI2010 simulation results with OCCAM KF

The turbulence model is described very detailed in Nilsson et al. (2007). Because it is time consuming to set up the fully populated variance-covariance matrix of observations, the approach of Nilsson has been modified to speed up the simulation. The correlation matrix is set up once and is then used for all 25 runs, only the random numbers are changing for each repetition and each station. The parameters which drive the simulation are the refractive index structure constant  $Cn$ , the wind speed and the height of the wet troposphere. In the following the influence of these three parameters on the results of the VLBI analysis is shown with respect to baseline length repeatabilities, 3 D station position and the change in station height.

For these studies, the turbulence parameters are chosen to be the same for all stations in the test network. The parameters not being varied, are set to standard values if not otherwise stated:

$Cn$ :  $1 \text{ e}^{-7} \text{ m}^{-1/3}$

Height: 2.0 km

Wind speed: 10 m/s towards East

For this investigation, the V2C 16 station test network and for scheduling the uniform sky algorithm with 60 seconds switching interval was used. The parameters of the schedule can be seen in Table 1. The parameters of the simulations of clock and  $wn$  values can be seen in Table 2 as well as general information about the analysis.

Table 1: Parameters of the uniform sky schedule used for the comparison of the turbulence parameters.

Station	switching intervall [sec]	uniform sky [min]	slew speed		number of observations
			az [°/sec]	el [°/sec]	
16	60	12	4.8	1.1	69 708

Table 2: Parameters of the simulation and analysis of the simulated data

clocks	ASD $1e^{-14}$ @ 50 min, random walk + integrated random walk
white noise	4 ps per baseline
software	OCCAM 6.2 SIM
variance rates for <i>zwd</i>	$0.7 \text{ ps}^2/\text{s}$
variance rates for gradients	$0.5 \text{ ps}^2/\text{s}$

## Changing the wind speed

Three different wind speeds were used, the wind was blowing towards East for all 3 simulations, with 5, 10 and 20 m/s:

Table 3: Median of the rms of station position for wind speeds of 5, 10 and 20 m/s.

wind [m/s]	5	10	20
median 3D rms [mm]	1.2	1.3	1.4
median Up rms [mm]	1.1	1.2	1.3
median North rms [mm]	0.3	0.4	0.4
median East rms [mm]	0.3	0.3	0.4

The baseline length repeatabilities show a dependence on the wind speed. The higher the wind speed, the faster the atmosphere over the station changes. If the atmosphere changes very fast, the separation of clocks and *zwd* is becoming harder and the repeatabilities are getting worse.

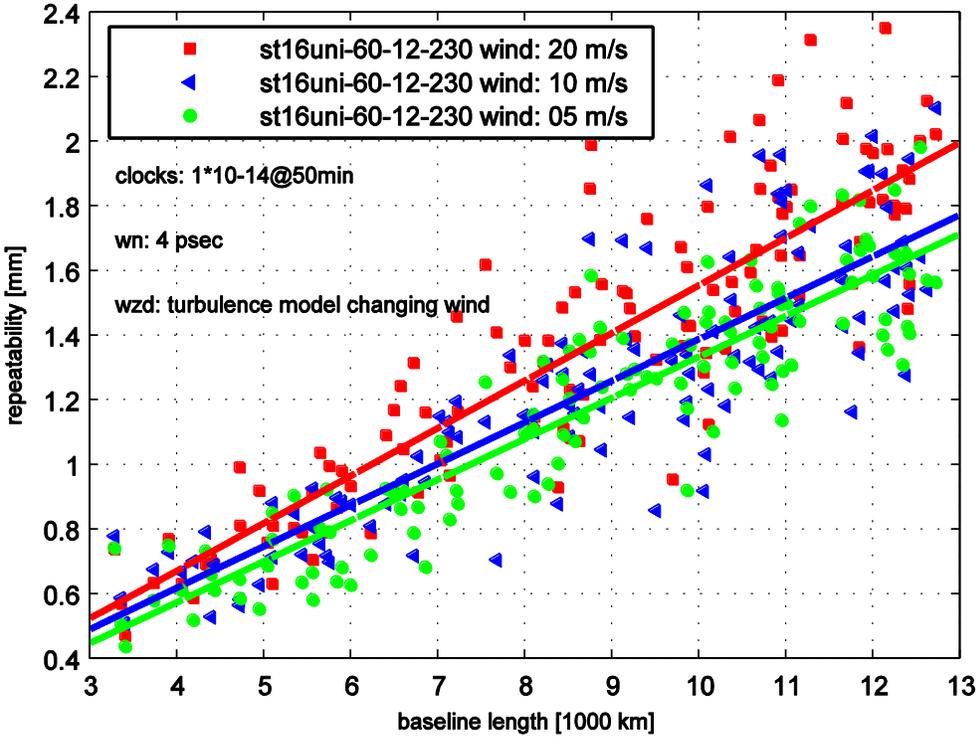


Figure 1: Baseline length repeatabilities for the 16 station uniform sky schedule with 60 seconds switching interval. For the simulation of the turbulent atmosphere, the wind speed was changed from 5, 10 to 20 m/s.

To quantify the performance of the simulation, the rms values and the median value of the 3D station position over all 16 stations are given in Figure 2. Also, the rms values and the medians of the vertical, North and East component of the station position are given in the figures below (Figure 3 – 5). The medians can be seen in Table 3. The error bars for the rms of the station positions are calculated as follows:

$$\sigma = \frac{rms}{\sqrt{2n}}$$

where  $n=25$  is the number of repetitions of the Monte Carlo Simulator.

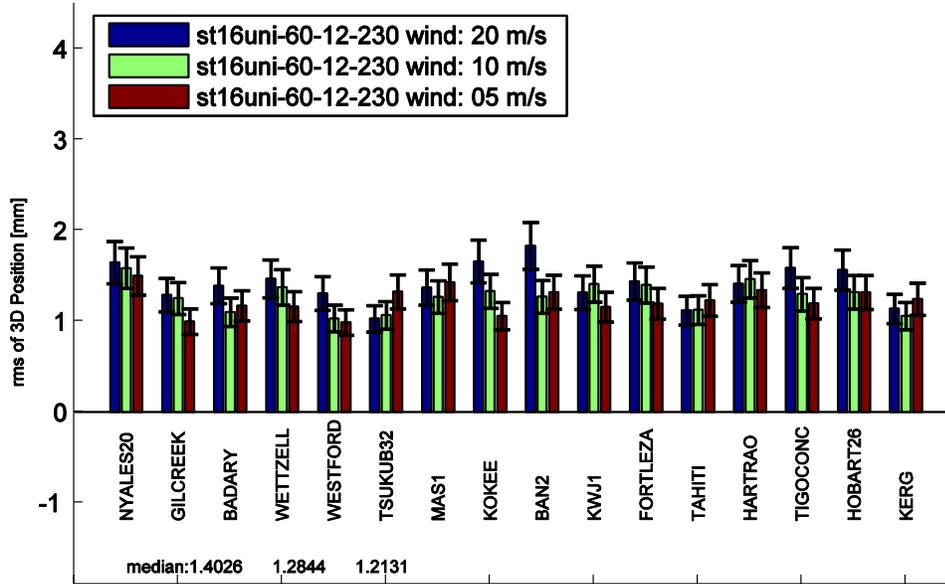


Figure 2: Rms of the 3D station position, the station order is North-South.

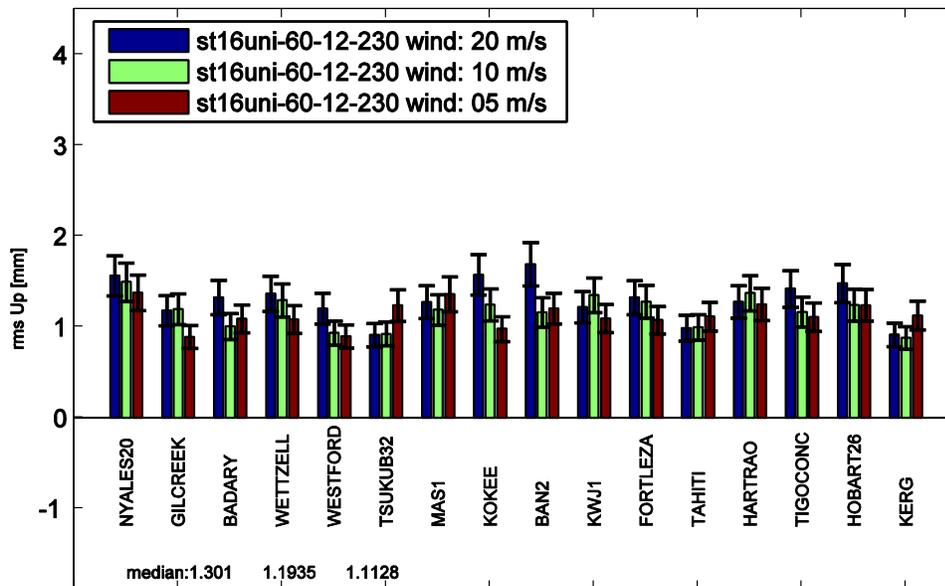


Figure 3: Rms of the vertical component of station position, the station order is North-South.

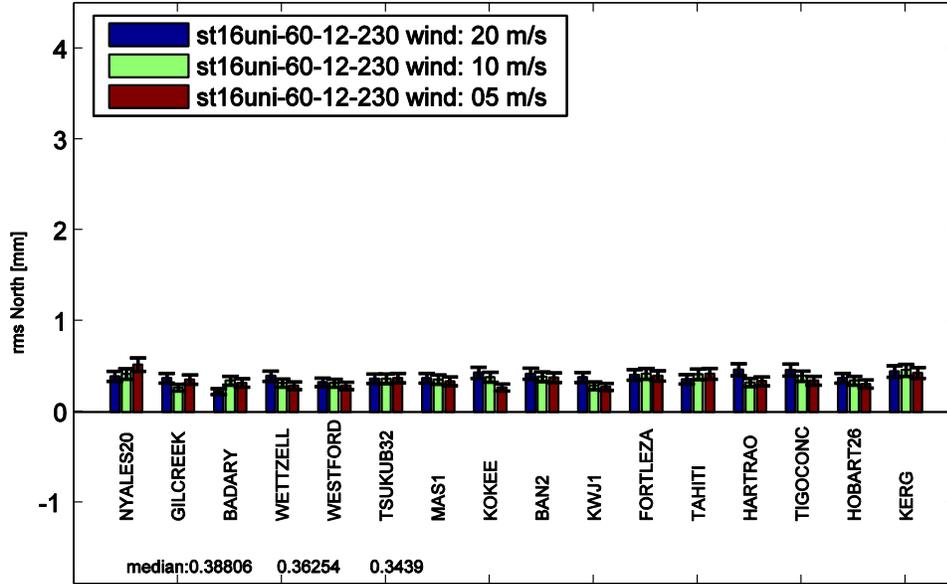


Figure 4: Rms of the North component of station position, the station order is North-South.

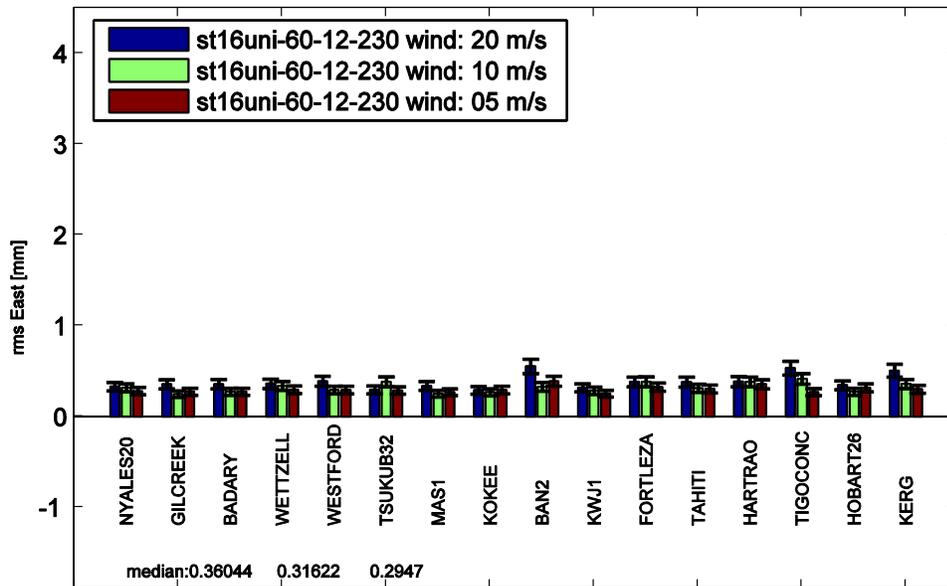


Figure 5: Rms of the East component of station position, the station order is North-South.

The behavior of the VLBI results on changing the wind speed for the simulation of the  $zwd$  changes to the opposite when the  $Cn$  and the height  $H$  change to a maximum, i.e. with high  $Cn$  and  $H$  values the rms of station positions improves with increasing wind speed. In Table 4 the medians of station position for different parameterizations of the  $zwd$  simulation are given.

Table 4: Median of the rms of station position for wind speeds of 5, 10 and 20 m/s, and for different  $Cn$  and  $H$  values.

wind [m/s]	5	10	20	5	10	20
$Cn e^{-7} [m^{-1/3}]$	1	1	1	3.5	3.5	3.5
$H$ [km]	2	2	2	4	4	4
median 3D rms [mm]	1.2	1.3	1.4	8.9	8.4	7.7
median Up rms [mm]	1.1	1.2	1.3	8.2	7.7	7.1
median North rms [mm]	0.3	0.4	0.4	2.9	2.3	2.1
median East rms [mm]	0.3	0.3	0.4	2.1	1.8	2.0

It can be seen that the effect on the VLBI analysis is reverse if we use very high values for  $Cn$  and  $H$ . The same can be seen in the studies of A. Pany with a PPP solution.

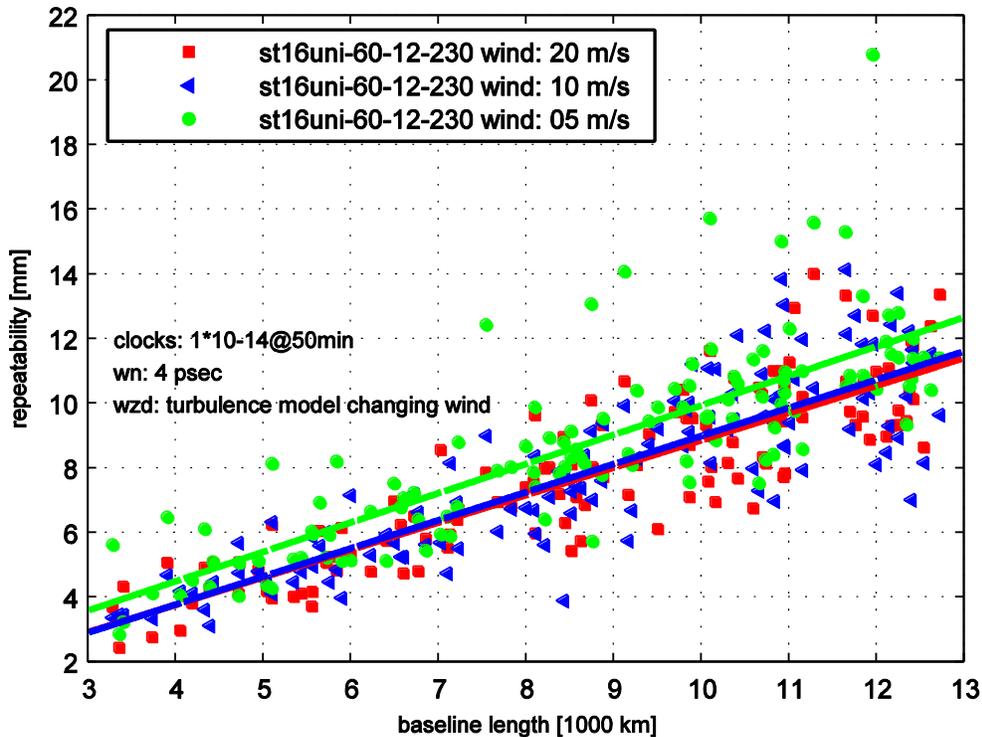


Figure 6: Baseline length repeatabilities for the 16 station uniform sky schedule with 60 seconds switching interval. For the simulation of the turbulent atmosphere, the  $Cn$  value is set to  $3.5 e^{-7} m^{-1/3}$  and the height to 4 km, only the wind speed was changed from 5, 10 to 20 m/s.

By looking at the 3D station position it can be seen that for the high  $Cn$  and  $H$  values and the low windspeed of 5 m/s the station Ny Ålesund is performing very poorly. The numbers of total baseline observations at the station Ny Ålesund is the same as for all other stations in the network. An effect of scheduling can be excluded. For the analysis with OCCAM, Ny Ålesund was picked as reference station, maybe this might have an effect for the very bad combination of the  $Cn$ ,  $H$  and wind speed values.

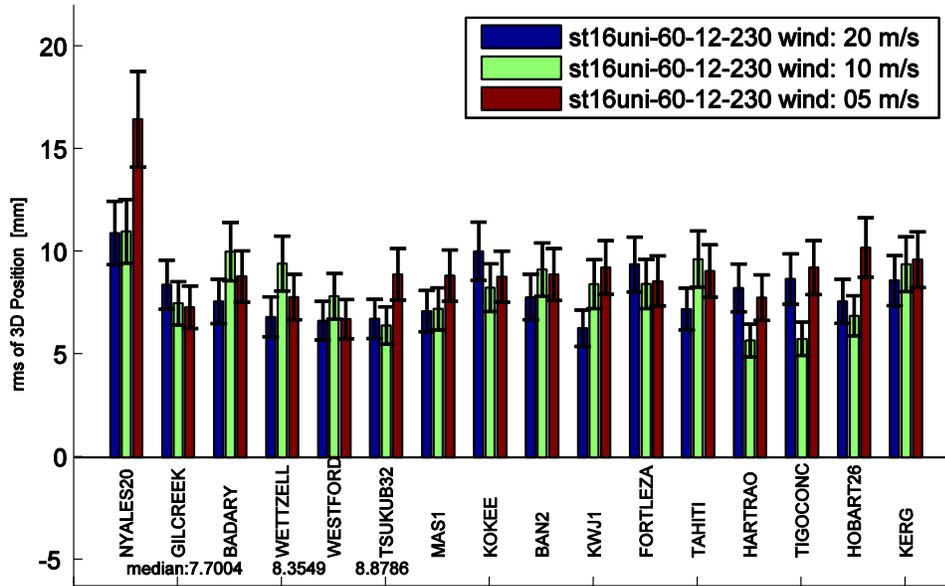


Figure 7: Rms of the 3D station position, the station order is North-South.

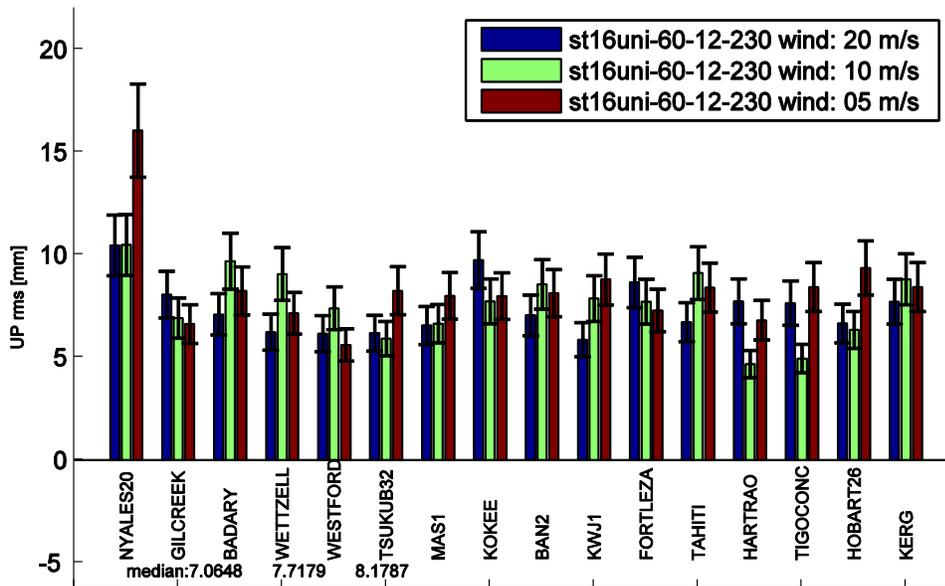


Figure 8: Rms of the vertical component of station position, the station order is North-South

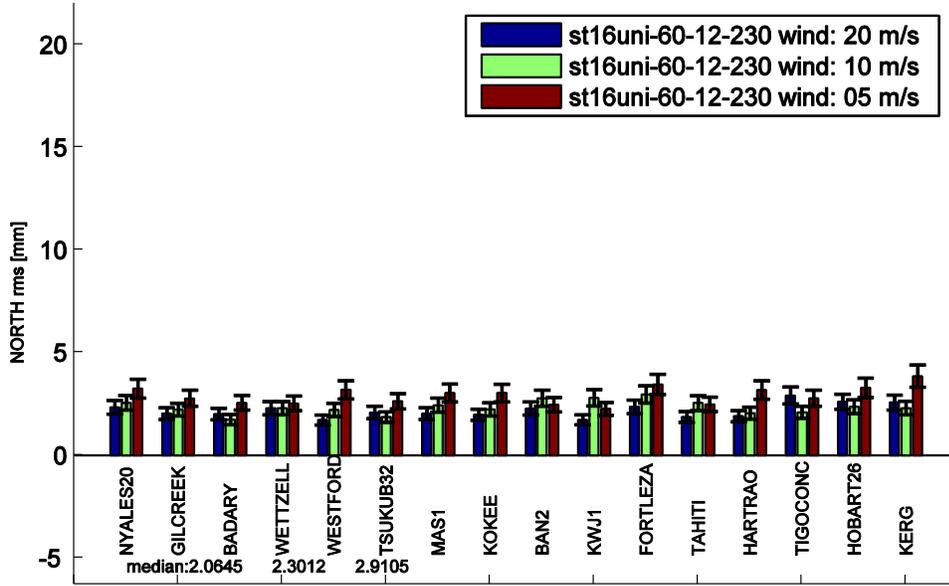


Figure 9: Rms of the North component of station position, the station order is North-South

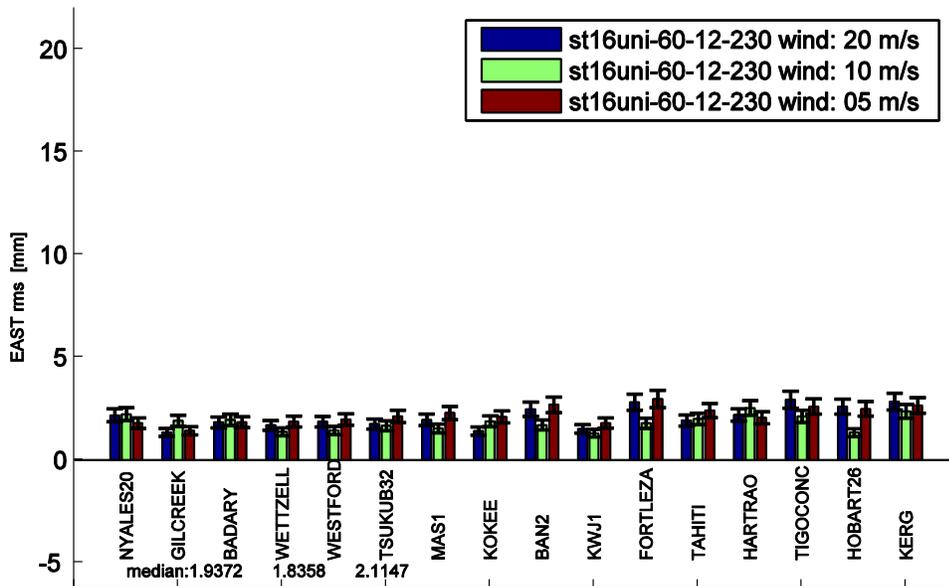


Figure 10: Rms of the East component of station position, the station order is North-South

For the simulation we have done already, the highest  $C_n$  value is  $2.5 \cdot 10^{-7} \text{ m}^{-1/3}$  and the maximum  $H$  is 2500 m. For a simulation with these  $C_n$  and  $H$  values (constant for all stations) the behavior of the VLBI parameters is shown in Table 5 compared to the previous results. The results show no big difference between the simulations using a wind speed of 5, 10 or 20 m/s. Thus a  $C_n$  of  $2.5 \cdot 10^{-7} \text{ m}^{-1/3}$  and a  $H$  of 2.5 km is somehow the threshold where the effect of the wind changes the behavior.

Table 5: Median of the rms of station position for wind speeds of 5, 10 and 20 m/s, , and for three different  $Cn$  and  $H$  values.

wind [m/s]	5	10	20	5	10	20	5	10	20
$Cn [e^{-7} m^{-1/3}]$	1	1	1	3.5	3.5	3.5	2.5	2.5	2.5
$H [km]$	2	2	2	4	4	4	2.5	2.5	2.5
median 3D rms [mm]	1.2	1.3	1.4	8.9	8.4	7.7	3.3	3.2	3.6
median Up rms [mm]	1.1	1.2	1.3	8.2	7.7	7.1	3.1	3.0	3.3
median North rms [mm]	0.3	0.4	0.4	2.9	2.3	2.1	0.9	0.9	0.9
median East rms [mm]	0.3	0.3	0.4	2.1	1.8	2.0	0.7	0.8	0.9

The influence of the wind speed for different  $Cn$  and  $H$  values is shown in Figure 11 which gives a summary of the median of the rms of 3D station position given in table 5.

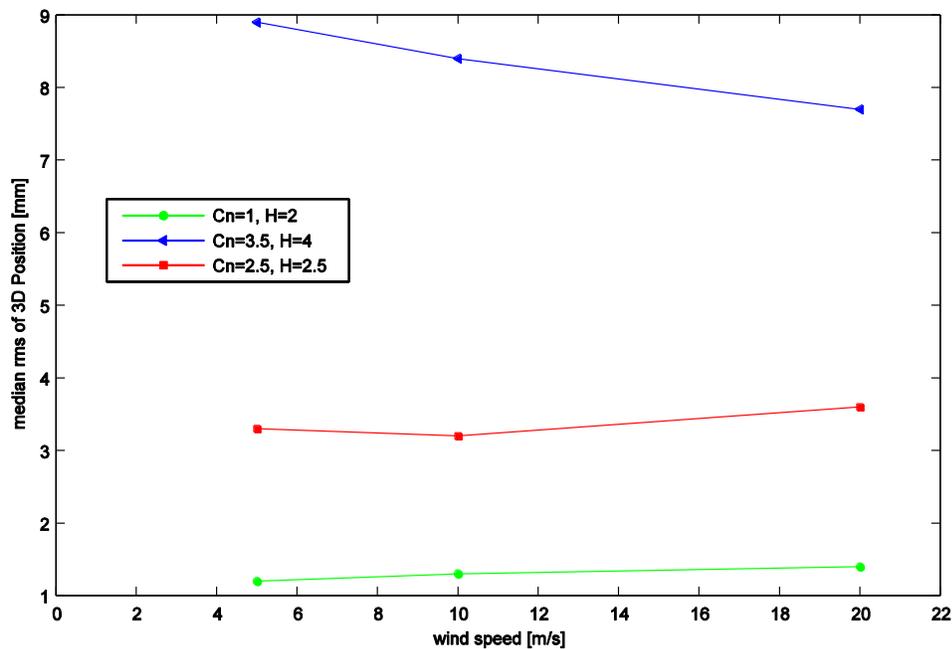


Figure 11: Median of the 3D station position for a simulation using 3 different wind speeds, 5, 10 and 20 m/s and 3 different combinations of  $Cn$  and  $H$  values. ( $Cn=1, H=2$ ;  $Cn=3.5, H=4$ ;  $Cn=2.5, H=2.5$ ).

## Changing the height of the wet troposphere

In the next step the height of the wet troposphere was changed from 1 to 2 and 3 km.

Table 6: Median of the rms of station position for different H of 1,2 and 3 km.

H[km]	1	2	3
median 3D rms [mm]	1.0	1.3	1.7
median Up rms [mm]	0.8	1.2	1.6
median North rms [mm]	0.2	0.4	0.5
median East rms [mm]	0.3	0.3	0.4

Figure 12 shows that the dependence of baseline length repeatability on the wet troposphere height is stronger than the dependence on different wind speeds (compare Figure 1). The higher the simulated height of the wet troposphere the larger is the baseline length repeatability.

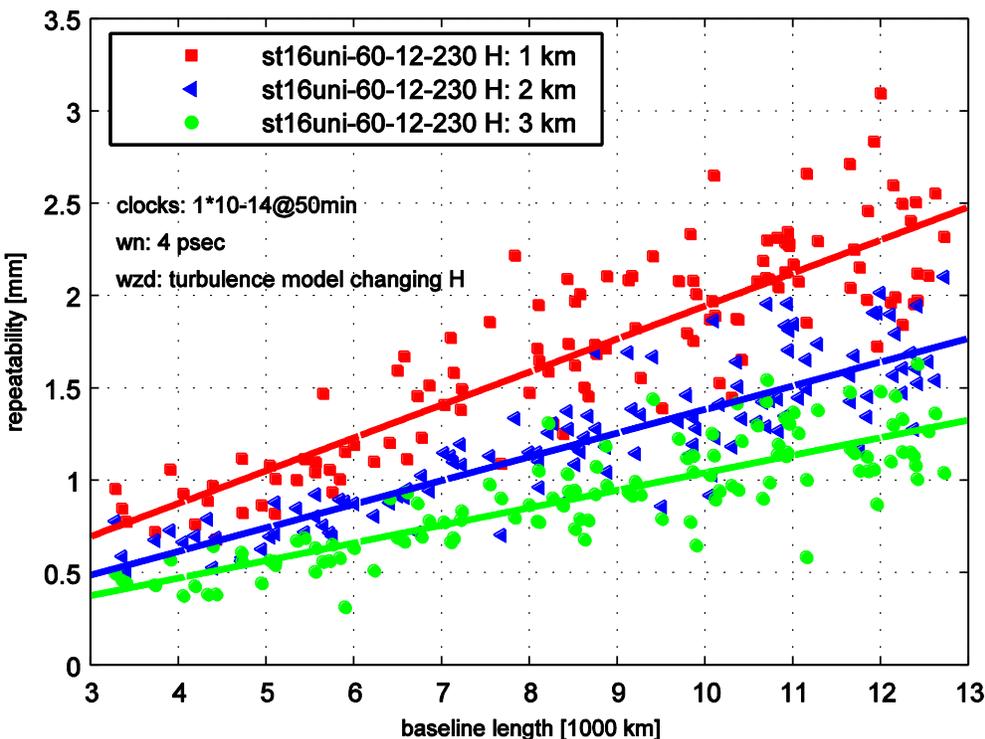


Figure 12: Baseline length repeatabilities for the 16 station uniform sky schedule with 60 seconds switching interval. For the simulation of the turbulent atmosphere the height of the wet troposphere was changed from 1, 2 to 3 km.

To quantify the performance of the simulation, the rms values and the median value for the 3D station position over all 16 stations are given in Figure 13. Also, the rms values and the median of the vertical, North and East component of the station position are given in the figures below (Figure 14 – 16). The medians can be seen in Table 6.

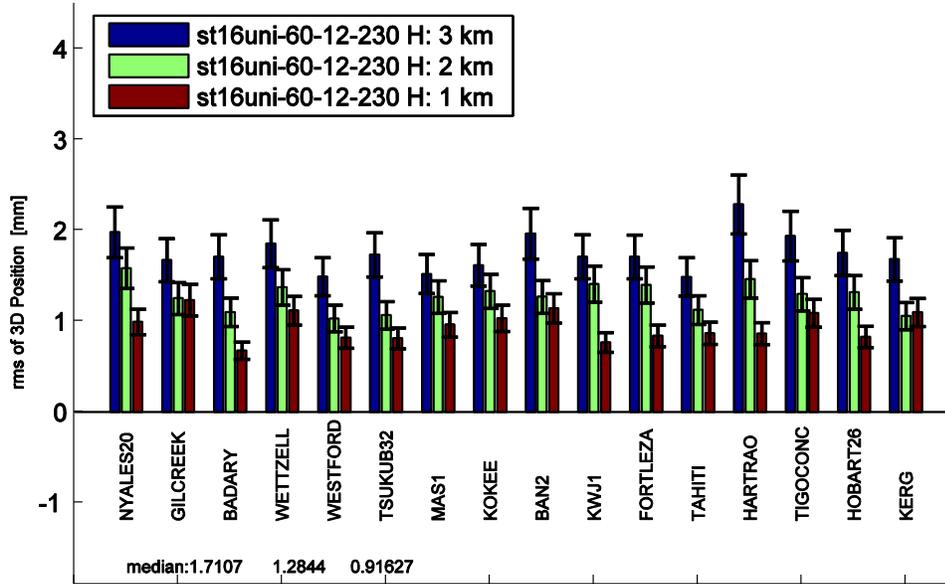


Figure 13: Rms of the 3D station position, the station order is North-South.

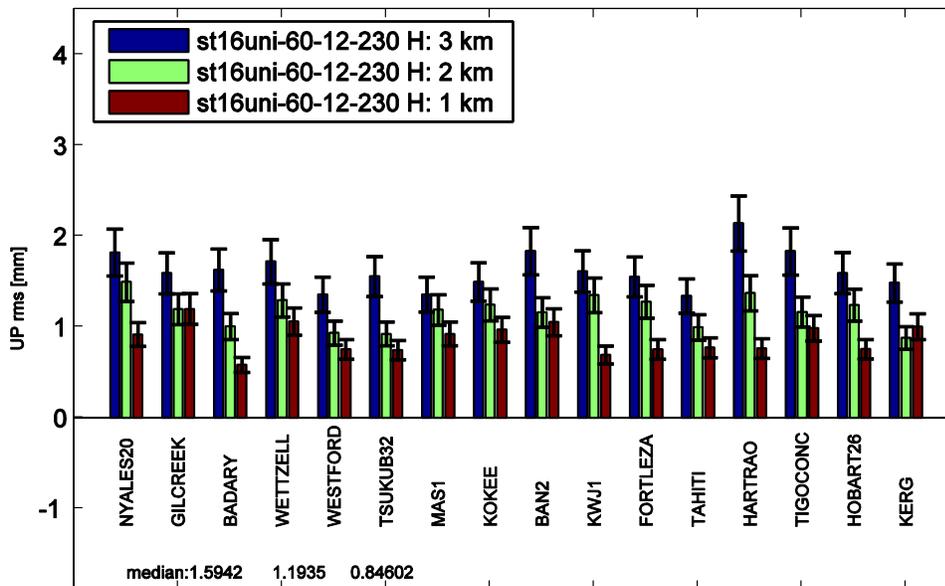


Figure 14: Rms of the vertical component of station position, the station order is North-South.

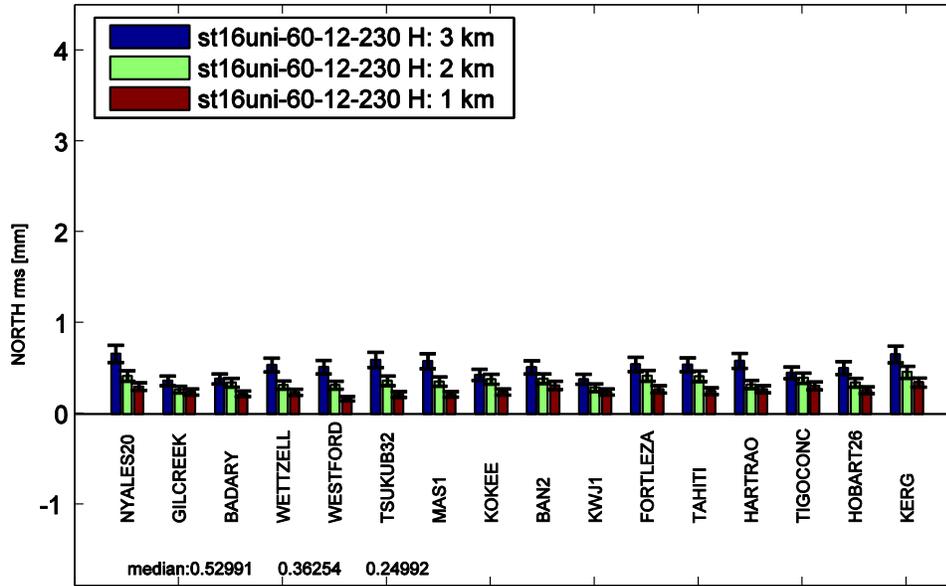


Figure 15: Rms of the North component of station position, the station order is North-South.

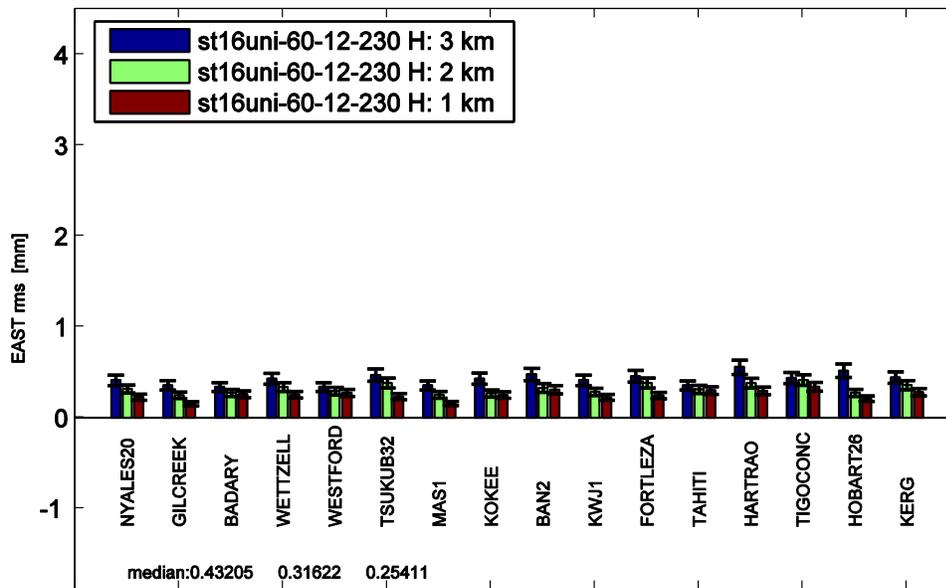


Figure 16: Rms of the East component of station position, the station order is North-South.

## Changing the Cn value

For the last comparison, the  $C_n$  value was changed.

Table 7: Median of the rms of station position for different  $C_n$  of 0.5, 1 and  $2 \cdot 10^{-7} \text{ m}^{-1/3}$ .

$C_n [e^{-7} \text{ m}^{-1/3}]$	0.5	1	2
median 3D rms [mm]	1.0	1.3	2.0
median Up rms [mm]	0.9	1.2	1.9
median North rms [mm]	0.2	0.4	0.5
median East rms [mm]	0.2	0.3	0.5

The change of the  $C_n$  values has a very high impact on the estimated parameters.

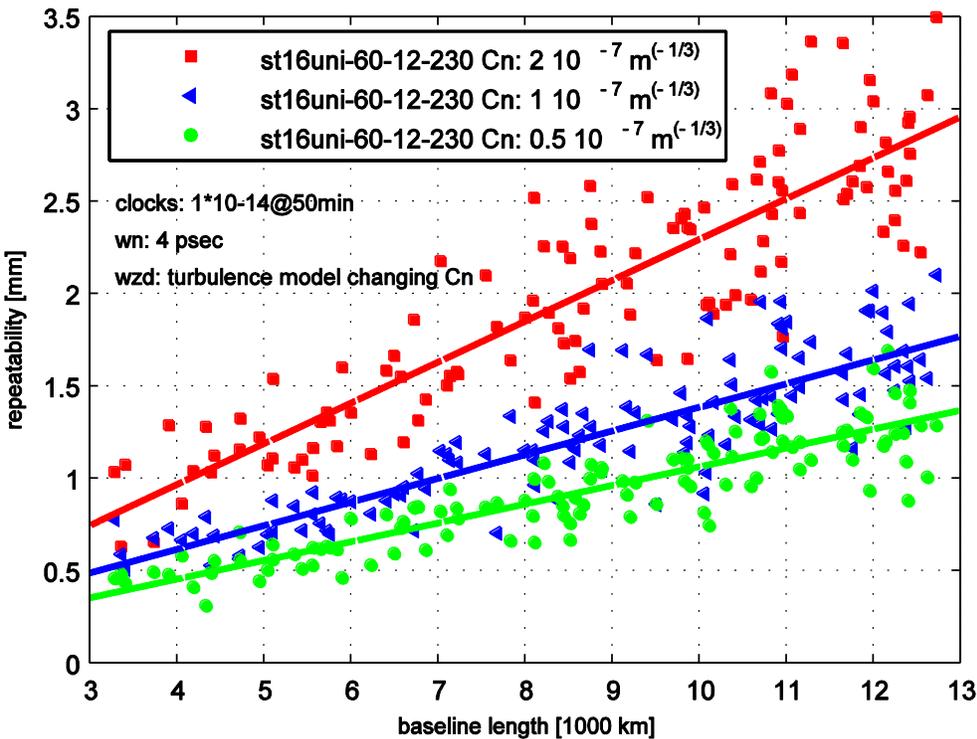


Figure 17: Baseline length repeatabilities for the 16 station uniform sky schedule with 60 seconds switching interval. For the simulation of the turbulent atmosphere the  $C_n$  values were changed from 0.5, 1 to  $2 \cdot 10^{-7} \text{ m}^{-1/3}$ .

To quantify the performance of the simulation, the rms values and the median value for the 3D station position over all 16 stations are given in Figure 18. Also the rms values and the median of the vertical, North and East component of the station position are given in the figures below (Figure 19 – 21). The medians can be seen in Table 7.

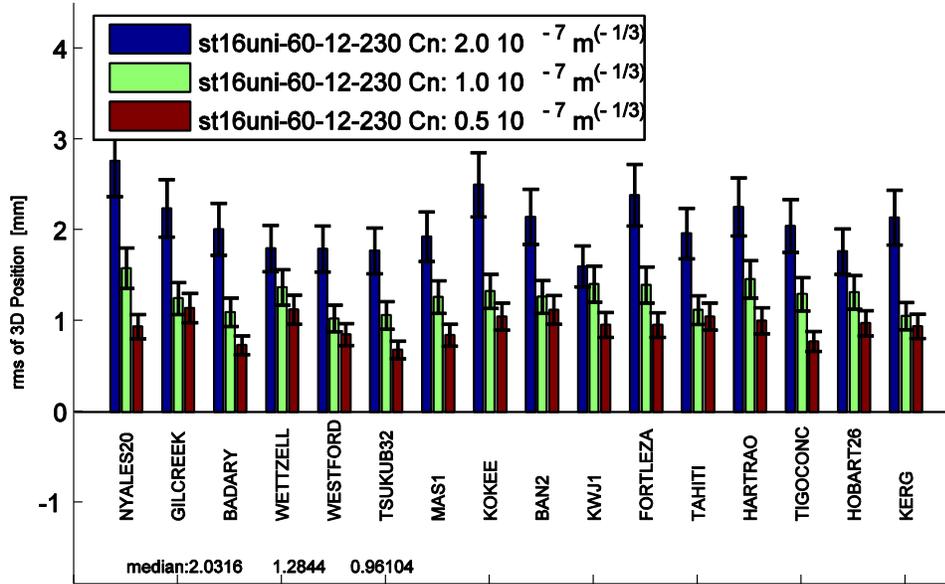


Figure 18: Rms of the 3D station position, the station order is North-South.

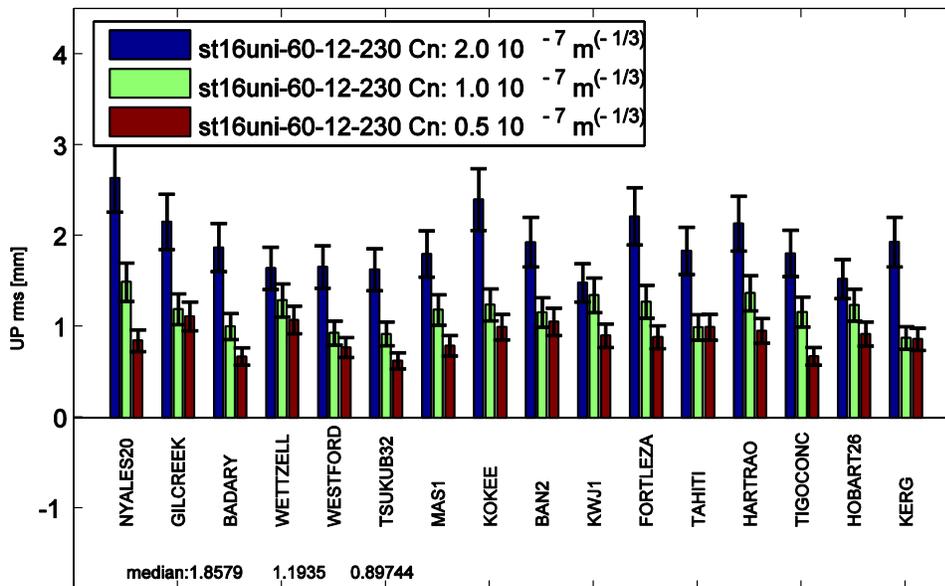


Figure 19: Rms of the vertical component of station position, the station order is North-South.

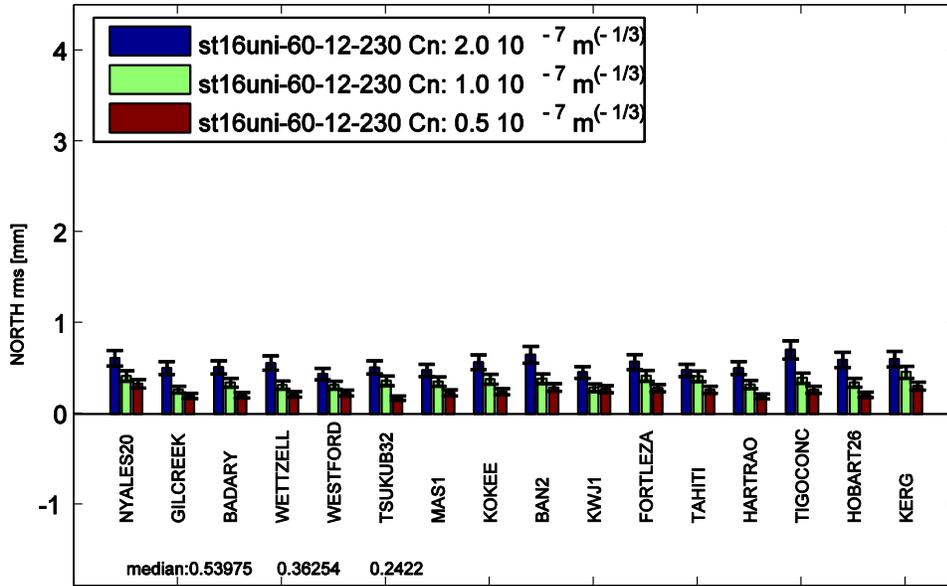


Figure 20: Rms of the North station position, the station order is North-South.

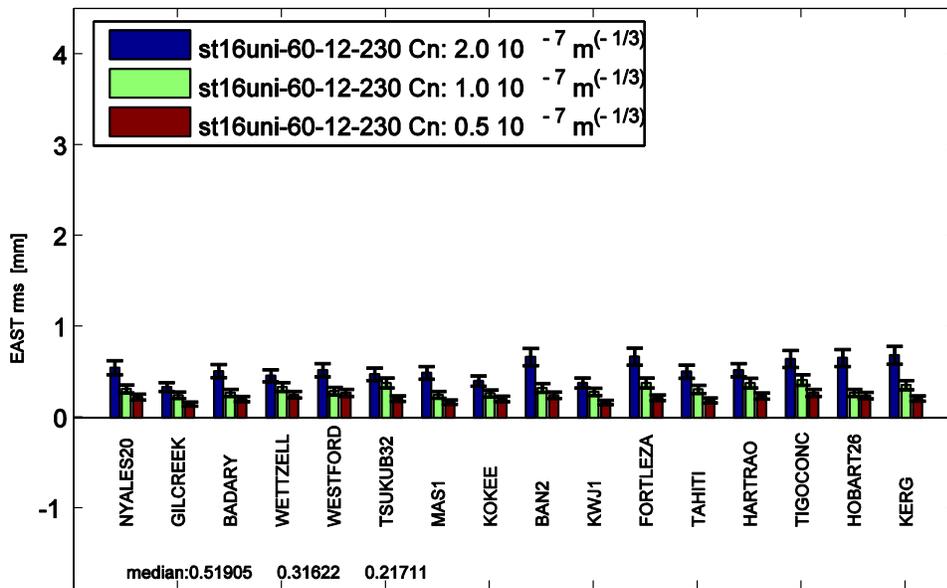


Figure 21: Rms of the East station position, the station order is North-South.

The summary plot given in figure 22 shows the change of  $C_n$  in black and the change of  $H$  in red. The median rms of the 3D station positions are listed in table 6 and 7.

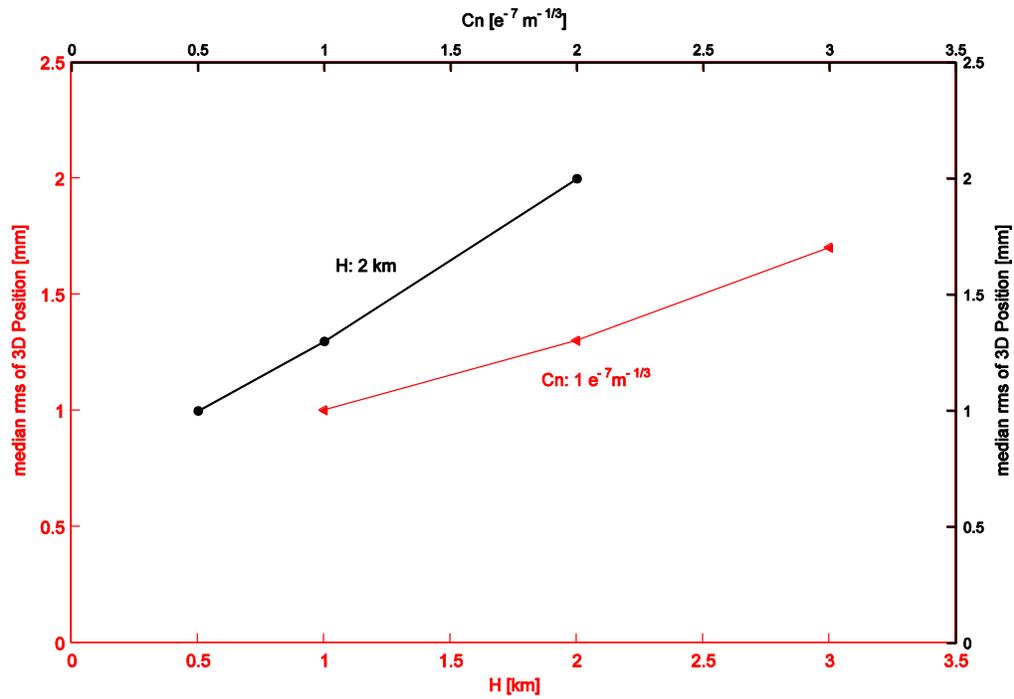


Figure 22: Median of the 3D station position for a simulation using 3 different  $C_n$ : 0.5, 1 and  $2 \text{ e}^{-7} \text{ m}^{-1/3}$  (black dots) and 3 different  $H$ : 1, 2 and 3 km (red triangles). The wind speed is 10 m/s.

## Impact of clock parameters on VLBI2010 simulation results with OCCAM KF

The clock is simulated as random walk plus integrated random walk (Böhm et al., 2007). The Allan standard deviation is changed to the values given in Table 8. For the v2c simulation, a white noise of 4 ps/bsl and a station dependent turbulence model were used. The parameters for  $Cn$  and  $H$  as well as the components of the wind vector in East and North directions used for the atmosphere simulations are listed in Table 9 for all 16 stations. The Vienna approach was used to simulate the zwd values. The evaluation and comparison with CONT05 real data for the used turbulence parameters is shown in Wresnik et al. (2008).

Table 8: Median of the rms of station position for simulated clocks with different ASD.

$ASD [ps^2/s]$	$5e^{-14}@50$	$2e^{-14}@50$	$1e^{-14}@50$	$5e^{-15}@50$	$1e^{-15}@50$
median 3D rms [mm]	3.9	2.1	1.5	1.4	1.3
median Up rms [mm]	3.6	1.9	1.4	1.3	1.2
median North rms [mm]	1.1	0.6	0.4	0.4	0.4
median East rms [mm]	1.0	0.5	0.4	0.3	0.3

Table 9: Station dependent  $Cn$ ,  $H$  and wind speeds for the turbulence model.

Station	$Cn$ [ $m^{-1/3}$ ]	$H$ [km]	$vnor$ [m/s]	$veas$ [m/s]
BADARY	0.86	1815	0.25	4.74
BAN2	2.47	1679	3.46	-2.2
FORTLEZA	2.47	2142	2.93	-7.12
GILCREEK	0.55	1963	3.8	-6.49
HARTRAO	2.03	1851	2.03	-2.84
HOBART26	1.16	2043	3.03	11.14
KERG	0.93	2089	3.4	17.5
KOKEE	2.3	1779	4.38	-3.36
KWJ1	2.47	1629	-1.64	-9.42
MAS1	2.47	1877	1.13	-4.87
NYALES20	0.35	1845	7.46	0.53
TAHITI	2.47	2078	5.45	-1.17
TIGOCONC	1.41	2176	1.21	4.96
TSUKUB32	1.45	1912	1.03	10.49
WESTFORD	1.17	2269	5.39	11.88
WETTZELL	0.94	1856	6.75	4.22

The v2c is using an ASD of  $1e^{-14}@50$ min for the simulation of the VLBI clocks. This simulates a realistic performance of an H-maser. Figure 23 shows the baseline length repeatabilities for different clock performances. Using a clock which is performing better than the v2c clock by a

factor of 10, is not improving the results significantly. This can also be seen in the 3D station position (figures 22-25)

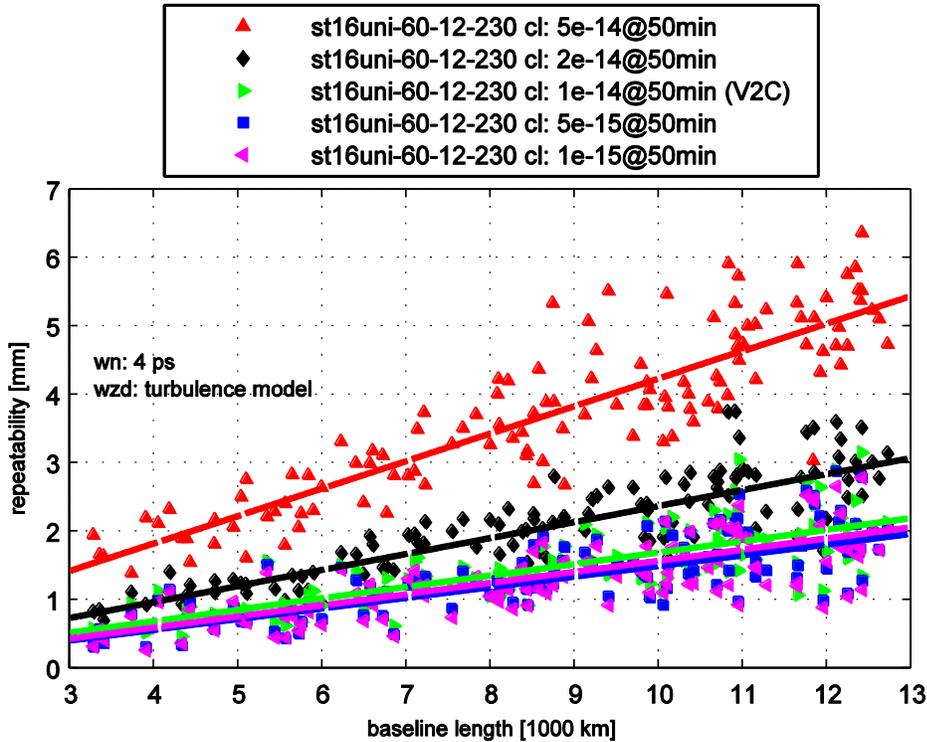


Figure 23: Baseline length repeatabilities for the 16 station uniform sky schedule with 60 seconds switching interval. For the simulation the clock values are set to  $1e^{-15}$ @50,  $5e^{-15}$ @50,  $1e^{-14}$ @50,  $2e^{-14}$ @50, and  $5e^{-14}$ @50 min.

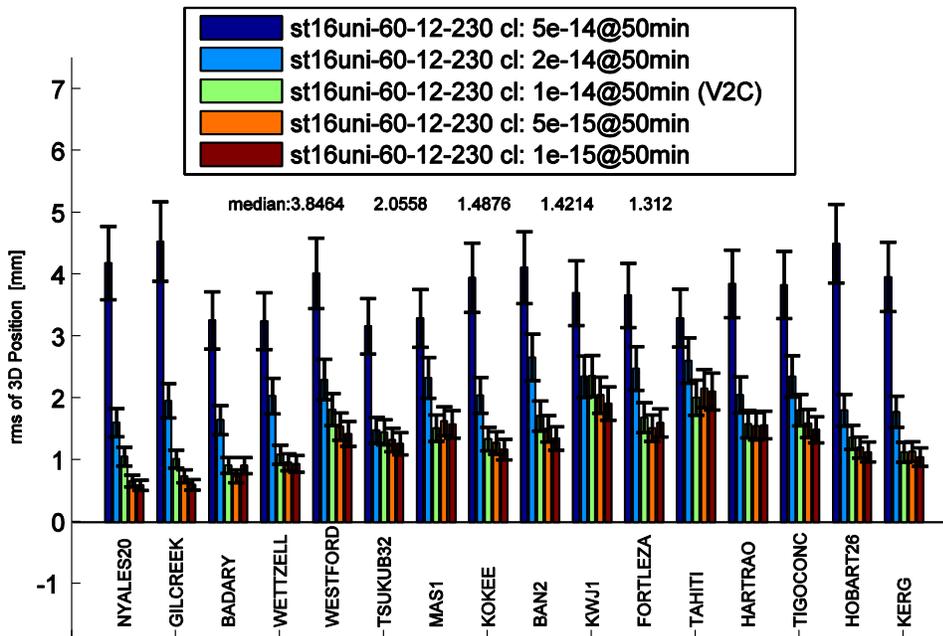


Figure 24: Rms of the 3D station position, the station order is North-South.

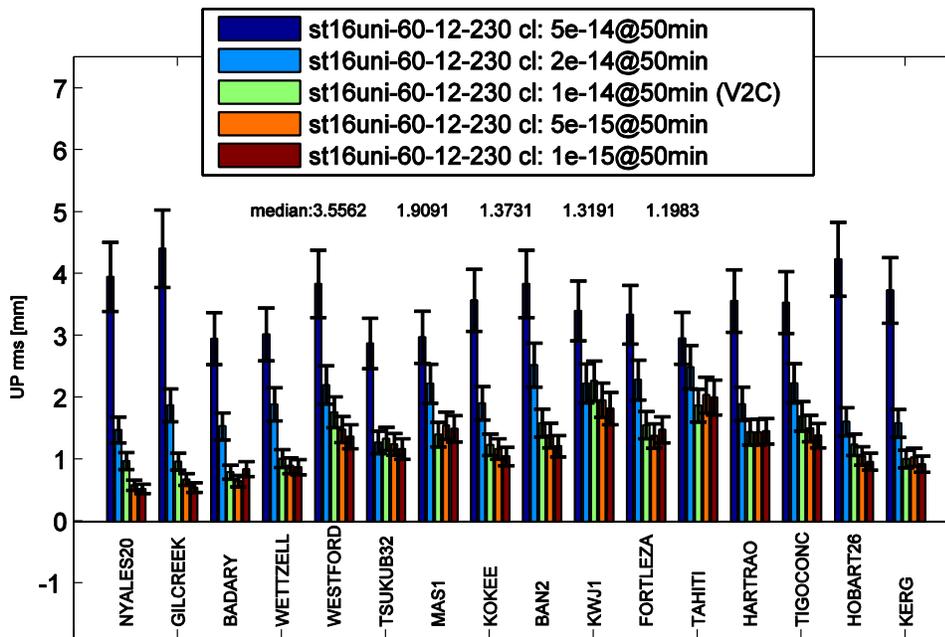


Figure 25: Rms of the vertical component of station position, the station order is North-South.

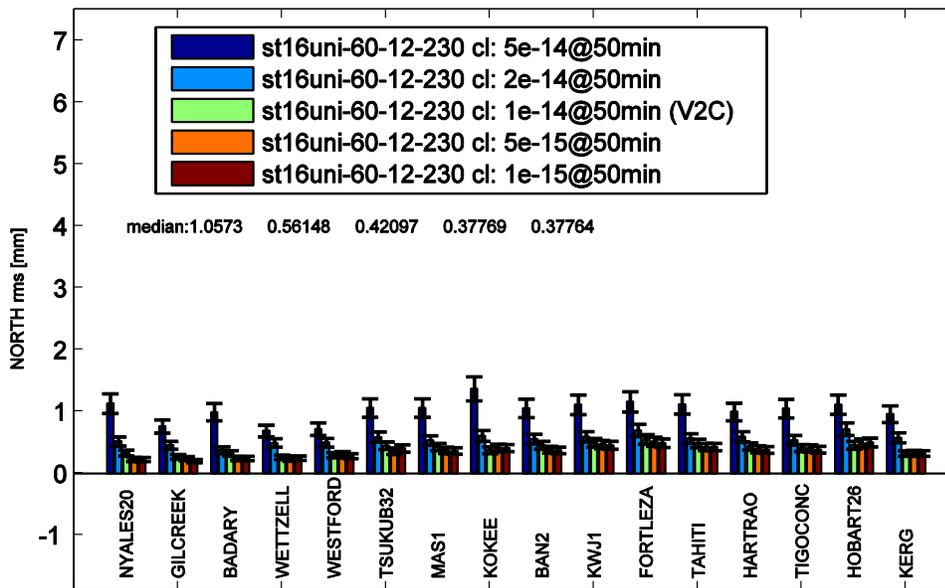


Figure 26: Rms of the North component of station position, the station order is North-South.

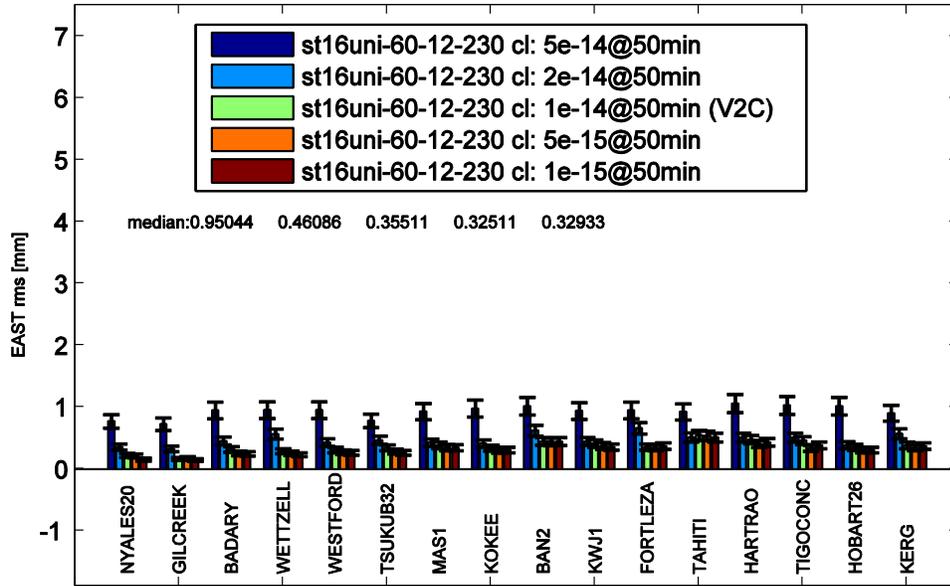


Figure 27: Rms of the East component of station position, the station order is North-South.

# Impact of added white noise on VLBI2010 simulation results with OCCAM KF

The standard value for the added white noise used for the v2c simulations is 4ps/bsl. This very low white noise models a perfect observing system with very small instrumental effects on the observation, which is the goal of VLBI2010. The today's accuracy of the VLBI system is represented by the simulation with 32 ps/bsl. The clocks are simulated with an ASD of  $1e^{-14}$ @50min and the zwd is simulated with the station dependent turbulence model, described above.

Table 10: Median of the rms of station position for simulations with different white noise.

<i>white noise [ps/bsl]</i>	4	8	12	16	24	32
median 3D rms [mm]	1.5	1.5	1.6	1.7	1.7	2.1
median Up rms [mm]	1.4	1.4	1.4	1.6	1.6	2.0
median North rms [mm]	0.4	0.4	0.4	0.4	0.6	0.6
median East rms [mm]	0.4	0.4	0.4	0.4	0.5	0.5

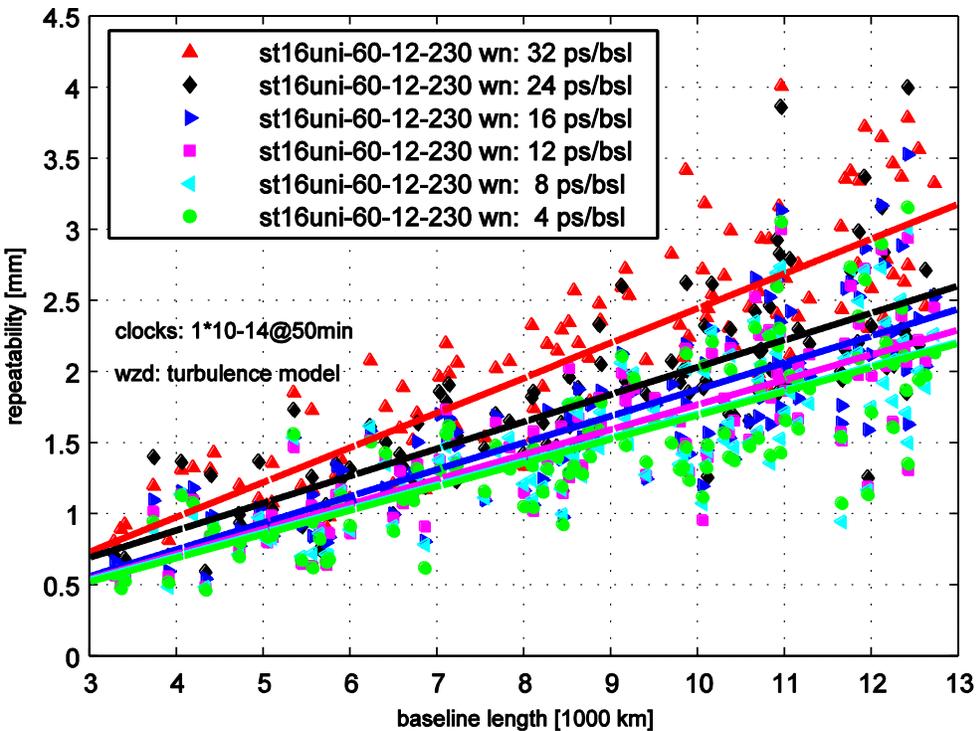


Figure 28: Baseline length repeatabilities for the 16 station uniform sky schedule with 60 seconds switching interval. For the simulation the white noise is set to 4, 8, 12, 16, 24, and 32 ps/bsl.

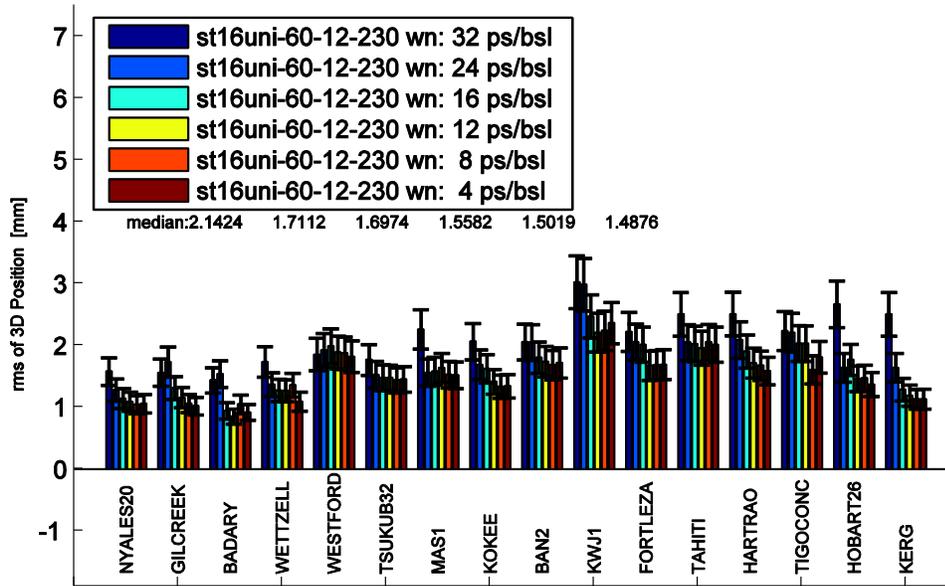


Figure 29: Rms of the 3D station position, the station order is North-South.

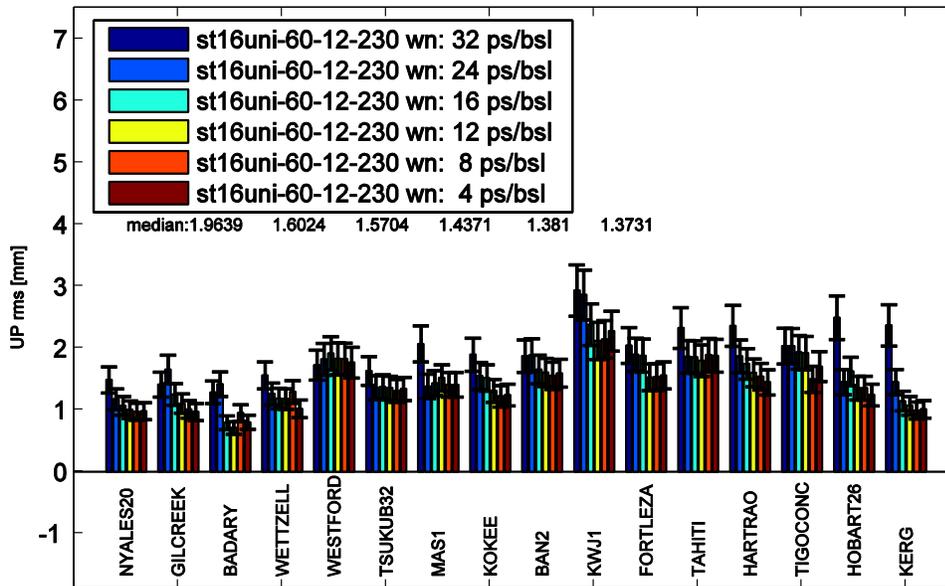


Figure 30: Rms of the vertical component of station position, the station order is North-South.

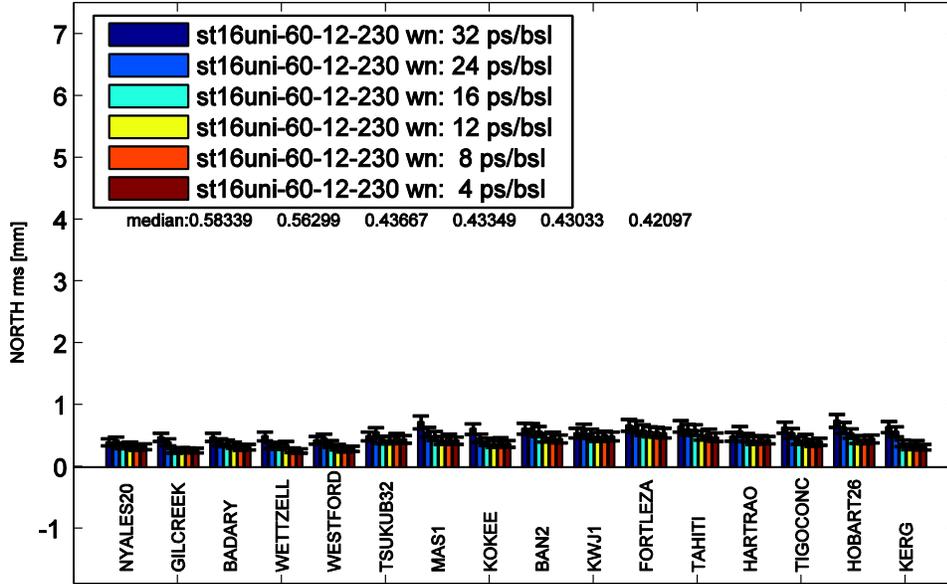


Figure 31: Rms of the North component of station position, the station order is North-South.

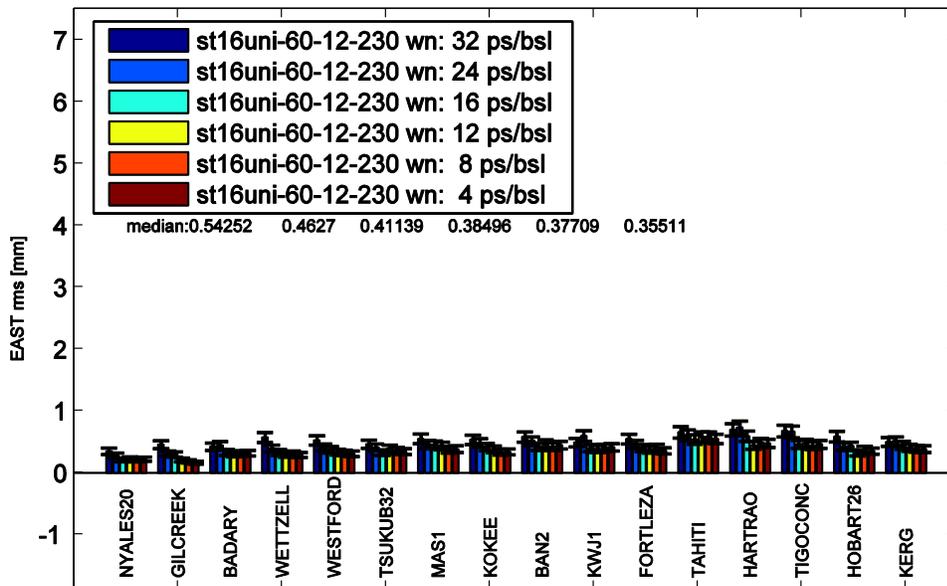


Figure 32: Rms of the EAST component of station position, the station order is North-South.

Figure 33 shows a summary of the simulation with different ASD for the clocks and with different white noise. The median rms of the 3D station positions are listed in table 8 and 10.

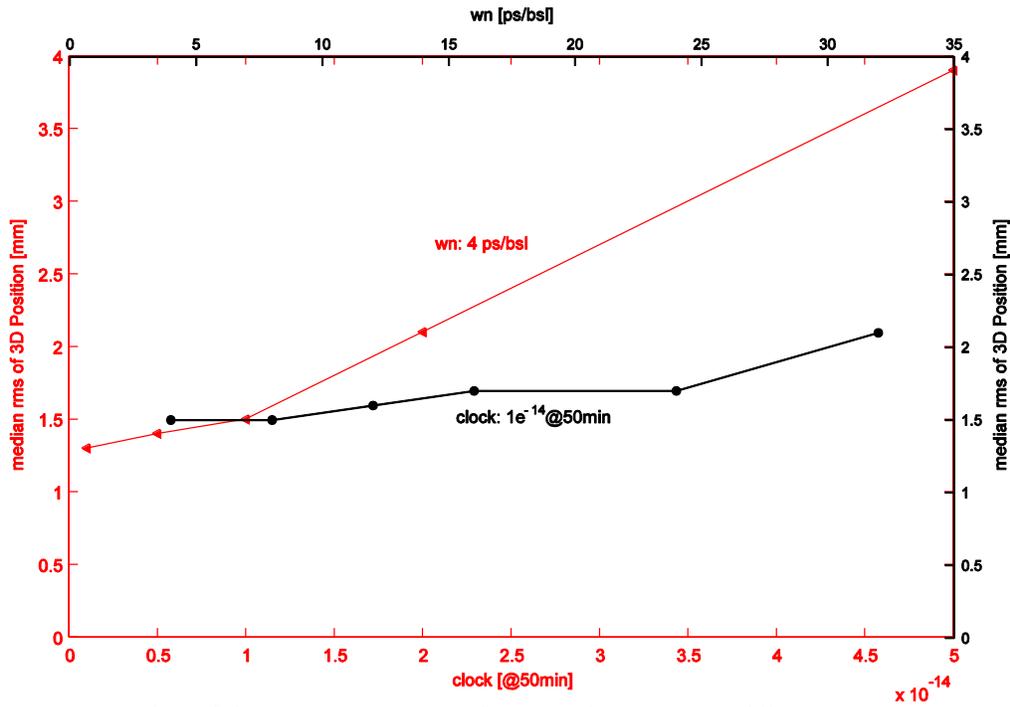


Figure 33: Median of the 3D station position for a simulation using 6 different wn: 4,8,12,16,24 and 32 ps/bsl (black dots, black axes) and 5 different ASD for the simulated clocks:  $1e^{-15}$ @50,  $5e^{-15}$ @50,  $1e^{-14}$ @50,  $2e^{-14}$ @50, and  $5e^{-14}$ @50 min (red triangles, red axes).

## Conclusions

From this investigation it can be concluded that the refractive index structure constant  $C_n$ , which is a measure for the "rockiness" of the troposphere, has the greatest impact on VLBI analyses. The second limiting factor concerning the troposphere is its effective height followed by the wind speed. For this investigation, when one of these three parameters is varied, the two others are set to the default values given above. It is to suppose that the variations in the results due to variation of one of the turbulence parameters will be the larger (or smaller) the larger (or smaller) the two other parameters are chosen. The impact of wind speed shows different behavior, depending on the choice of  $C_n$  and  $H$ : For low  $C_n$  and  $H$ , the repeatabilities degrade with higher wind speed, for high  $C_n$  and  $H$ , the repeatabilities improve with higher wind speed.

Assuming that

- the turbulence model yields a realistic description of the troposphere, and
- the schedule used for this investigation can be achieved by the new VLBI2010 system,

it can be stated that the 1 mm target of VLBI2010 might be hard to realize as long as the troposphere models are not significantly improved.

Looking at the simulated clocks with an ASD of  $1e^{-14}$ @50 min it can be concluded, that the used value is realistic and that a further improvement of the clock to an ASD of  $1e^{-15}$ @50 doesn't yield an improvement of the VLBI analysis.

Concerning the simulation of the instrumental error with an added white noise per baseline, the proposed value of the  $v_2c$  is very optimistic and assumes a perfect observing system, which will be very hard to realize. The simulation with different levels of white noise shows a big improvement from a  $w_n$  of 32 ps/bsl (which simulates the today's standard in VLBI) to 8 ps/bsl. No improvement can be seen between a  $w_n$  of 8 and 4 ps/bsl. This might be due to the strong influence of the atmosphere on the VLBI observations.

## References

Boehm J., J. Wresnik, A. Pany, Simulation of wet zenith delays and clocks, IVS Memorandum 2006-013v03, <http://ivscc.gsfc.nasa.gov/publications/memos/index.html> (2007)

Nilsson, T., R. Haas, G. Elgered, Simulations of atmospheric path delays using turbulence models, In: Proceedings of the 18th European VLBI for Geodesy and Astrometry Working Meeting, 12-13 April 2007, edited by J. Böhm, A. Pany and H. Schuh, Geowissenschaftliche Mitteilungen, Heft Nr. 79, Schriftenreihe der Studienrichtung Vermessung und Geoinformation, Technische Universität Wien, ISSN 1811-8380, 175-180, (2007)

Wresnik J., A. Pany, J. Boehm, Evaluation of the new  $C_n$  values for the turbulence model with CONT05 real data, IVS Memorandum 2008-004v01, <http://ivscc.gsfc.nasa.gov/publications/memos/index.html> (2008)